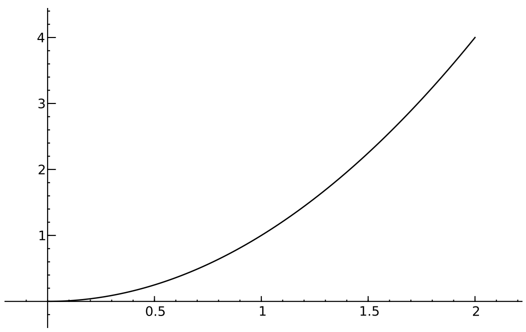
WORKSHEET FOR 2/2/2009

Reading assignment for Friday. Read section 7.3.
Homework due Friday. 7.2: 6, 11, 14, 19, 38, 39

Theorem 1. *A solid lies with its base in the xy-plane between the vertical planes $x = a$ and $x = b$. For all x in $[a, b]$, let $A(x)$ denote the area of the cross section at x perpendicular to the x-axis. If $A(x)$ is a continuous function, then*

$$\textit{Volume} = \int_a^b A(x)dx$$

(1) Write down an expression for the length of the curve $y = \sin(e^x)$ on the interval $[3, 4]$. You do not need to evaluate the integral.



(2) Let V be the volume of the solid obtained by rotating about the x-axis the region R bounded above by the curve $y = f(x)$, below by the x-axis, on the left by the vertical line $x = a$, and on the right by the vertical line $x = b$. Furthermore, suppose that $a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$ and that $x_{j-1} \leq t_j \leq x_j$ for all j. This breaks up the line into n intervals, and we'll be sampling the function at the points t_j.
   (a) Explain why $V \approx \sum_{j=1}^{n} \pi(f(t_j))^2(x_j - x_{j-1})$, using theorem 1.
   (b) Explain why $V = \int_a^b \pi(f(x))^2 dx$.
   (c) Suppose that $f(x) = x^2$, $a = 0$ and $b = 2$ Find V.

(3) Let V be the volume of the solid whose base is the region bounded by $y = x^2$ and the x-axis (same region as in 2(c)) and whose cross-section at x is one of the following shapes. Find V in the following cases:
   (a) A square with side length x^2.
   (b) An equilateral triangle with side length x^2.
   (c) A semi-circle with radius $x^2/2$.