WORKSHEET FOR 2/6/2009

Reading assignment for Monday. Read section 6.3.
Homework due Monday. 7.3: 2, 5, 6, 10, 11. The following website has some neat graphical animations of how to do work problems, which might be helpful on your homework:
http://archives.math.utk.edu/visual.calculus/5/work.1/index.html
Finding work in today's job market: Recall that work is force times distance. For example, the work done moving a five pound weight from the floor to a height of four feet is (5 lb)(4 ft) = 20 ft $\cdot\, lb$ (pounds are a measure of force). In this case, it doesn't matter if we first lift the weight to five feet, then lower it to four–the work done is the same, since the force required to lower the weight has the opposite sign from the force required to raise. The two forces cancel out to yield the same answer. The subtlety comes when the force changes with distance. For example, it's actually true that the force required to raise the weight decreases slightly as you raise it (since the weight gets farther from the center of the Earth). We define the work done by a force function $F(x)$ from a to b as $W = \int_a^b F(x)dx$.

(1) Springs approximately follow Hooke's law: the force exerted by a string is proportional to the distance from its rest position or natural length. In symbols: $F(x) = kx$, where k is a fixed constant that depends on the spring. Suppose that a spring has a natural length of 18in, and that a force of 10 lb is enough to compress it to a length of 16 in.
  (a) What is the value of the spring constant k.
  (b) How much work is done in compressing the spring from 16 to 12in? How much work is done expanding the spring from 18 to 20in.

(2) Sometimes it's difficult to compute the force function just by looking at the problem. In these cases, Riemann sums can be your friends.[1]
Say we've got a tank made by rotating the curve $y = x^2$ on the interval $[0, 4]$ around the y-axis. Suppose that the tank is filled with a liquid with a density ρ pounds per cubic foot. Let's figure out how much work would be required to pump all the liquid over the top of the tank.
  (a) First let's "slice" the tank at height $y = y_i$, with thickness Δy. Then we can approximate the volume in this slice by a cylinder with volume $\pi(\sqrt{y})^2\Delta y = \pi y \Delta y$ (where does $\sqrt{y}$ come from?). How much work is done moving this to the top of the tank?
  (b) Write a Riemann sum approximating how much work is done moving all the liquid to the top of the tank.
  (c) Write an expression for exactly how much work is done moving all the liquid to the top of the tank, and evaluate this expression.

(3) Find the amount of work to move a tank up to the top of a building which is 250 feet high. The tank is 5 foot high and weighs 200 pounds. A cable is attached to the top of the tank and a winch on the top of the building; it weighs 3 pounds per foot.

[1] Riemann sums are really nice guys, if you give them a chance. They'd totally help you move into a new apartment. Or at least figure out how much work would be required to move all your stuff to your new place.