WORKSHEET FOR 2/11/2009

Reading assignment for Wednesday. Read section 8.1. For business, finance or economics majors, section 7.5 is recommended
Homework due Friday. 7.4: 17, 18, 20, 28, 29, 30
Notes: We call a differential equation *separable* if it is of the form $y' = \frac{dy}{dt} = f(t)g(y)$ (note that this includes the cases $\frac{dy}{dt} = f(t)/g(y)$ and $\frac{dy}{dt} = g(y)/f(t)$). In this case, we can write "separate" variables as follows:

$$\begin{aligned} \frac{dy}{dt} &= f(t)g(y) \\ \frac{dy}{g(y)} &= f(t)dt \\ \int \frac{dy}{g(y)} &= \int f(t)dt \end{aligned}$$

Example 1: Last time, we looked at the initial value problem (IVP) $\frac{dy}{dt} = y^2$, $y(1) = -1$. Separating variables gives:

$$\begin{aligned} \frac{dy}{dt} &= y^2 \\ \frac{dy}{y^2} &= dt \\ \int \frac{dy}{y^2} &= \int dt \\ \frac{-1}{y} + C_1 &= t + C_2 \\ \frac{-1}{y} &= t + C \quad \text{(where } C = C_2 - C_1\text{)} \\ y &= \frac{-1}{t+C} \end{aligned}$$

Now we can use the initial value to solve for C. If $y(1) = -1 = \frac{-1}{1+C}$, then clearly $C = 0$.
Example 2: Consider the IVP $\frac{dy}{dt} = (y-2)\cos t$, $y(0) = 1$. Separating variables gives that $\int \frac{dy}{y-2} = \int \cos t dt$. Thus $\ln|y-2| + C_1 = \sin t + C_2$, so $y - 2 = \pm e^{\sin t + C}$, so $y = 2 \pm e^{\sin t + C}$. Setting $1 = 2 \pm e^{\sin 0 + C}$, we see that $-1 = \pm e^C$, which only has solution $-1 = -e^0$ (in particular $C = 0$). Thus $y(t) = 2 - e^{\sin t}$ is the exact solution to our IVP. To check that our answer is correct, all we need to do is check the initial value: $y(0) = 2 - e^{\sin 0} = 2 - 1 = 1$, and check that the differential equation is satisfied: $y' = -(\cos t)e^{\sin t} = (\cos t)((2 - e^{\sin t} - 2) = (\cos t)(y - 2)$.

(1) Find a solution to the differential equation $x' = 1 + x^2$. Check that your solution is correct.
(2) Find a solution to the initial value problem $y' = \frac{t}{y}$, with $y(1) = 3$. Check that your solution is correct.
(3) Suppose that y is a function of x satisfying $y' - xy^2 = 0$, and $y(1) = 1$. Find y and check that your answer is correct. Use Euler's method with 4 steps to estimate $y(2)$. What is the error of approximation in this case?