WORKSHEET FOR 2/13/2009

Reading assignment for Monday. Read section 8.2.
Homework due Monday. 8.1: 6, 21, 30, 53, 54, 65abc
Notes: Recall the product rule:

$$\frac{d}{dx}(u \cdot v) = u \cdot v' + u' \cdot v$$

We can integrate with respect to x both sides to obtain:

$$\begin{aligned} \int (\frac{d}{dx} u \cdot v) dx &= \int u \cdot v' dx + \int u' \cdot v dx \\ u \cdot v &= \int u \cdot v' dx + \int u' \cdot v dx \\ u \cdot v - \int u' \cdot v dx &= \int u \cdot v' dx \\ u \cdot v - \int v du &= \int u dv \end{aligned}$$

This yields the integration by parts formulas: $\int u dv = u \cdot v - \int v du$ and $\int_a^b u dv = [uv]_a^b - \int_a^b v du$

Example: $\int_0^\pi x \sin x dx$. Finding u and dv is the hardest part of doing these problems. At first you have to take some guesses and see what works. As you do more problems, you'll gain some intuition for good choices. In fact, all you need to do is find u, and dv is just everything else. Here a good guess is $u = x$ and $dv = \sin x$, so $v = -\cos x$:

$$\begin{aligned} \int_0^\pi x \sin x dx &= [x \sin x]_0^\pi - \int_0^\pi \sin x dx \\ &= (-\pi \cos \pi + 0 \cos 0) - [-\cos x]_0^\pi = \pi - (-\cos \pi + \cos 0) = \pi \end{aligned}$$

(1) Evaluate $\int x f''(x) dx$.
(2) Find $\int x^r \ln x dx$, where $r \neq 0$.
(3) Evaluate $\int_0^\pi x^2 \cos x dx$.
(4) (a) Show that $\int \sin^2 x dx = -\sin x \cos x + \int \cos^2 x dx$.
 (b) Use part (a) and the identity $\sin^2 x + \cos^2 x = 1$ to show that $\int \sin^2 x dx = \frac{1}{2}(x - \sin x \cos x) + C$.