WORKSHEET FOR 2/16/2009

Reading assignment for Wednesday. Read sections 8.3, 8.4 (8.4 is very short)
Homework due Wednesday. 8.2: 25, 26, 29, 44, 45, 46
Notes: Long division for polynomials works just like long division for numbers. See http://www.youtube.com/watch?v=l6_ghhd7kwQ for a good tutorial.
Given an integral of the form $\int \frac{p(x)}{q(x)} dx$, where p and q are polynomials, we use the following steps:

- Use long division to write $p(x)/q(x) = s(x) + r(x)/q(x)$, where s and r are polynomials, and r has degree less than q.
- Factorize q, and decompose $r(x)/q(x)$ into its partial fraction decomposition.
- Do the (now much simpler) integral.

Example: $\int \frac{3x^3+2x^2+2x+1}{x^2-1} dx$. Long division gives that $\frac{3x^3+2x^2+2x+1}{x^2-1} = 3x + 2 + \frac{5x+3}{x^2-1}$. We factorize $x^2 - 1 = (x-1)(x+1)$. We need to solve the equation:

$$\frac{5x+3}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{(A+B)x + (A-B)}{(x-1)(x+1)}.$$

This gives us two equations in two unknowns $A + B = 5$, $A - B = 3$, which we can solve in the usual way: Add second equation to first $2A + B - B = 8$, so $A = 4$. Then $B = 1$. Thus:

$$\begin{aligned}\int \frac{3x^3+2x^2+2x+1}{x^2-1} dx &= \int (3x+2)dx + \int \frac{4}{x-1} dx + \int \frac{1}{x+1} dx \\ &= \frac{3}{2}x^2 + 2x + 4\ln|x-1| + \ln|x+1| + C\end{aligned}$$

(1) Consider $\int \frac{4x^3+21x^2+31x+3}{x^2+5x+6}$.
   (a) Use long division to divide $\frac{4x^3+21x^2+31x+3}{x^2+5x+6}$.
   (b) Factorize $x^2 + 5x + 6$.
   (c) Use your answers from parts (a) and (b) to find the partial fraction decomposition.
   (d) Evaluate the integral.
(2) Evaluate the integral $\int \frac{x^2+5x+8}{x(x^2+4x+6)} dx$. (Hints: Since the numerator has lower degree than the denominator, you just need to do the partial factions decomposition. One term will be easy to integrate, for the other, complete the square in the denominator, and do a substitution.)
(3) Evaluate the integral $\int \frac{5x^3-3x^2+x+1}{x-1} dx$