WORKSHEET FOR 2/18/2009

Reading assignment for Wednesday. Read section 9.1.
Homework due Wednesday. 8.3: 33, 26, 27, 65; Extra credit (5pts): 59, 64 (Hint for 59: use a right triangle with one angle $x/2$ for parts (a) and (b).)
Notes: Useful identities:

(1) $\sin^2 x + \cos^2 x = 1$
(2) $\tan^2 x + 1 = \sec^2 x$
(3) $\sin^2 x = \frac{1}{2} - \frac{\cos(2x)}{2}$
(4) $\cos^2 x = \frac{1}{2} + \frac{\cos(2x)}{2}$.
(5) $\sin 2x = 2 \sin x \cos x$

The idea in the second half of 8.3 is to use u-substitution "in reverse" to simplify certain kinds of integrals. This is a trick, which can seem a bit weird until you've done some examples. **Example:** $\int \sqrt{1-x^2}dx$
If we substitute $x = \sin t$, then $dx = \cos t dt$. Then we have:

$$\begin{aligned}
\int \sqrt{1-x^2}dx &= \int \sqrt{1-\sin^2 t}\cos t dt \\
&= \int \sqrt{\cos^2 t}\cos t dt \\
&= \int |\cos t|\cos t dt \\
&= \int \cos^2 t dt \quad \text{If } \cos t \geq 0, \text{ which is true if } -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \\
&= \int (\frac{1}{2} + \frac{\cos(2t)}{2})dt \\
&= \frac{t}{2} + \frac{\sin 2t}{4} + C \\
&= \frac{\arcsin x}{2} + \frac{1}{4} 2 \sin t \cos t \\
&= \frac{\arcsin x}{2} + \frac{1}{2} x\sqrt{1-\sin^2 t} \\
&= \frac{\arcsin x}{2} + \frac{1}{2} x\sqrt{1-x^2}
\end{aligned}$$

Look at the table at the top of p.480.

(1) Calculate $\int \frac{1}{\sqrt{1-x^2}}dx$ by substituting $x = \sin t$.
(2) Calculate $\int \sqrt{x^2+4}$. (Hint: look at the table on p.480.)
(3) Calculate $\int \frac{1}{\sqrt{-4x^2+8x-3}}$ by completing the square.