WORKSHEET FOR 3/10/2009

Reading assignment for Friday. Read section 10.3.
Homework due Friday. 11.2: 5, 14, 18, 27, 28, 29 (use 27 in 28 & 29), 63

Notes: Our goal is to make sense of infinite sums like $\sum_{k=1}^{\infty} a_k$. The trick is to look at the sequence of partial sums: $S_n = \sum_{k=1}^{n} a_k$. Then we define:

$$\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum_{k=1}^{n} a_k.$$

An important kind of infinite series are *geometric series*, i.e. the series of the form $\sum_{k=0}^{\infty} ar^k$, $r \neq 1$. The partial sums satisfy the formula:

$$\sum_{k=0}^{n} ar^k = a \cdot \frac{1-r^{n+1}}{1-r}, \text{ and so for } |r| < 1$$

$$\sum_{k=0}^{\infty} ar^k = \lim_{n\to\infty} \sum_{k=0}^{n} ar^k = \lim_{n\to\infty} a \cdot \frac{1-r^{n+1}}{1-r} = \frac{a}{1-r}$$

Geometric sums diverge for $|r| \geq 1$.
An important theorem for partial sums is the following theorem on monotone sequences:

Theorem. *Every bounded monotone sequence converges.*

This theorem can allow us to show a series converges, even if we can't find its exact value. (See problem 1).

(1) Suppose that for all n, $a_k \geq 0$, and the partial sums $\sum_{k=1}^{n} a_k \leq c$.
   (a) Show that $\{S_n\}$ is a monotone sequence.
   (b) Show that $\{S_n\}$ is bounded. (Hint: a_k is nonnegative for all k.)
   (c) Show that $\sum_{k=1}^{\infty} a_k$ converges.

(2) Explain why $\sum_{k=0}^{\infty} (-1)^n$ does or does not converge by evaluating the partial sums.

(3) Evaluate the following series:
   (a) $\sum_{n=0}^{\infty} e^{-n}$.
   (b) $\sum_{j=1}^{\infty} 2^{-n}$. Note that j starts at 1, not 0.
   (c) $\sum_{k=1}^{\infty} k$.
   (d) $\sum_{k=0}^{\infty} 2^n$.

(e) $\displaystyle\sum_{m=0}^{\infty} \frac{3^m + 7^m}{6^m}$

(f) $\displaystyle\sum_{i=0}^{\infty} \sin i$

(g) $\displaystyle\sum_{l=1}^{\infty} \sin(2\pi l)$

(h) $\displaystyle\sum_{p=0}^{\infty} \cos(\pi p)$