WORKSHEET FOR 3/16/2009

Reading assignment for Wednesday. 11.4
Homework due Wednesday. 11.3: 22, 24, 26, 28, 30
Notes: The ratio test is a useful modification of the comparison test.

Theorem. *(Ratio test for positive series) Suppose that $a_k > 0$ for all k, and that*

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = L$$

- *If $L < 1$, then $\sum a_k$ converges.*
- *If $L > 1$, then $\sum a_k$ diverges.*
- *If $L = 1$, then the test is inconclusive. The series could diverge or diverge.*

Example 1: Consider $a_k = 1/k!$. We showed with some difficulty in lab last week that $\displaystyle\sum_{k=0}^{\infty} \frac{1}{k!} = e$, so we know it converges. Using the ratio test, we can show this much more easily:

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim \frac{1/(k+1)!}{1/k!} = \lim \frac{k!}{(k+1)!} = \lim \frac{1}{k+1} = 0 < 1$$

Thus by the ratio test, the series converges.
Example 2: Consider $a_k = 1/k^2$. By this time, we've showed in two different ways that the corresponding series converges. However, the ratio test is no help:

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim \frac{1/(k+1)^2}{1/k^2} = \lim \frac{k^2}{(k+1)^2} = 1$$

Thus the ratio test tells us nothing about whether this series converges.
Exercises:

(1) Use the ratio test to show that the following series converge:
   (a) $\displaystyle\sum_{j=0}^{\infty} \frac{j^2}{j!}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$
   (c) $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!}$, where $x > 0$.

(2) (a) What, if anything, does the ratio test say about the convergence of the series $\frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \cdots$?
   (b) Does the series in part (a) converge? Justify your answer.

(3) Determine whether the following series converge or diverge. Use whatever method you wish.
   (a) $\displaystyle\sum_{j=0}^{\infty} \frac{1}{j^2+5}$.
   (b) $\displaystyle\sum_{m=1}^{\infty} \frac{m^3}{m^5+3}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n3^n}$
   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{(2n)!}$