

HOMEWORK 10 TO BE GRADED

- (1) [10 points] Consider the Boundary Value Problem with Neumann BCs on the interval $[0, \pi]$

$$u_t = k u_{xx}, \quad 0 < x < \pi, \quad t > 0,$$

$$u_x(0, t) = u_x(\pi, t) = 0, \quad t > 0,$$

$$u(x, 0) = f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \quad 0 < x < \pi.$$

Here, k is a positive constant and a_n are the Fourier cosine coefficients of $f(x)$, i.e.,
$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx.$$

By separation of variables, show that the solution of the above BVP is

$$u(x, t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \exp(-kn^2t) \cos nx.$$

Also, find the limit of $u(x, t)$ as t tends to ∞ .

- (2) [5 points each] Solve problems 7, 10, 14 in section 9.5.