MATH 385-X1
DIFFERENTIAL EQUATIONS:
TEST 2

FALL, 2004

Name : ______________________

Formulas

Here are some formulas you might be able to use on the test.

- $mx'' + cx' + kx = 0$
- Given $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Given $ax^2 + 2bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - ac}}{a}$ (Note that the linear term is $2bx$ instead of bx here.)
- $$y_p = y_1 \int \frac{-y_2 f}{W(y_1, y_2)}\, dx + y_2 \int \frac{y_1 f}{W(y_1, y_2)}\, dx, \quad W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_1' y_2$$

(1) Find the general solution of the following homogeneous linear differential equation with constant coefficients (Hint: You might use that $y = e^x$ is a solution of the DE): [6 pts]

$$y^{(3)} + 4y'' + y' - 6y = 0.$$

(2) Find the appropriate form of a particular solution of the following nonhomogeneous linear differential equation with constant coefficients. You do not need to determine the coefficients: [6 pts]

$$y^{(4)} + y^{(3)} - 2y'' = 3e^{2x} + 4x + 1.$$

(3) Find a particular solution of the following differential equation: [6 pts]

$$y'' + 9y = \sec 3x.$$

(4) Suppose that the mass in an unforced mass-spring-dashpot system with $m = 1$, $c = 2$ and $k = 5$ is set in motion with $x(0) = 1$ and $x'(0) = -1 + 2\sqrt{3}$.

(a) Find the position function $x(t)$. [7 pts]

(b) Find the pseudoperiod. [1 pt]

(c) Find the equations of the envelope curves. [2 pts]

(d) Determine the behavior of $x(t)$ (f.e., decays, grows, oscillates, decays while oscillating or grows while oscillating). [1 pt]

(e) Find the time lag. [**Extra Credit of 2 pts**]

(5) Find all non-negative eigenvalues and an associated eigenfunction for each eigenvalue of the following boundary value problem (Determine whether zero is an eigenvalue. If so, write an eigenfunction associated with zero. And then find all positive eigenvalues with eigenfunctions): [8 pts]

$$y'' + \lambda y = 0, \; y'(0) = 0, \; y'(1) = 0.$$

For **Extra Credit of 3 pts**, show that there is no negative eigenvalue for the given boundary value problem.

(6) Consider the following forced undamped oscillator equation:

$$x'' + 4x = \cos \omega t.$$

(a) Solve the given differential equation (Hint: You must consider two separate cases). [10 pts]

(b) Determine the behavior of the solution $x(t)$ for $\omega = 1$ and $\omega = 2$ (f.e., decays, grows, oscillates, decays while oscillating or grows while oscillating). [2 pts]
$\omega = 1$:

$\omega = 2$:

(c) Determine the value of ω when the resonance occurs. [1 pt]