

Problem 1

The doomsday vs. extinction model

$$x' = ax\left(\frac{x}{T} - 1\right)$$

may need to be modified so that unbounded growth does not occur then x is above the threshold $T > 0$. The simplest way to do this is to introduce another factor that will have the effect of making $\frac{dx}{dt}$ negative when $x > K > T$. Thus, we consider

$$x' = ax\left(\frac{x}{T} - 1\right)\left(1 - \frac{x}{K}\right).$$

1. Find the equilibria and draw the phase line for this equation.
2. Sketch some representative solutions for initial conditions $0 < x(0) < T$, $T < x(0) < K$, and $x(0) > K$. (Use the slope field of the equation.)
3. Repeat the previous part for $K = T$.

Problem 2

It is sometimes reasonable to assume that the rate at which fish are caught depends on their population x : the more fish there are, the easier it is to catch them. To include this effect the logistic equation is replaced by

$$x' = ax\left(1 - \frac{x}{K}\right) - Ex.$$

1. Find the two equilibria $x_1 < x_2$ for this equation if $E < a$.
2. A sustainable yield $x_s = Ex_2$ of the fishery is a rate at which fish can be caught indefinitely. Find x_s as a function of E .
3. Determine E so as to maximize x_s and thereby find the maximum sustainable yield x_m .

Problem 3

For each of the following differential equations, find all equilibrium solutions and determine whether they are sinks, sources, or shunts. Also, sketch the phase line.

$$(i) \quad x' = x^3 - 4x; \quad (ii) \quad x' = |1 - x^2|.$$

Problem 4

Each of the following families of differential equations depends on a parameter a . Sketch the corresponding bifurcation diagrams.

$$(i) \quad x' = x^3 - ax; \quad (ii) \quad x' = x^3 - x - a.$$