MATH 489, Section C14, HW 1. Due date: 02/06/09.

Problem 1

Some diseases– such as typhoid fever– are spread largely by carriers, individuals who can transmit the disease but who exhibit no symptoms. Let, x and y, respectively, denote the proportion of susceptibles and carriers in the population. Suppose that carriers are identified and removed from the population at a rate $\beta > 0$

$$y' = -\beta y.$$

Suppose also that the disease spreads at a rate proportional to the product of x and y, thus

$$x' = -\alpha xy.$$

1. Determine y at time t.
2. Use the result of the previous part to find x at any time t.
3. Find the proportion of the population that escapes the epidemic by finding the limiting value of x as $t \to \infty$.

Problem 2

Consider the family of differential equations

$$x' = ax + \sin x$$

where a is a parameter.

1. Sketch the phase line when $a = 0$.
2. Use the graphs of ax and $\sin x$ to determine the qualitative behavior of all the bifurcations that occur as a increases from -1 to 1.
3. Sketch the bifurcation diagram for this family of differential equations.

Problem 3

Consider the first order nonautonomous equation

$$x' = p(t)x,$$

where $p(t)$ is differentiable and periodic with period T. Prove that all solutions of this equation are periodic with period T if and only if

$$\int_0^T p(s)ds = 0.$$