

Problem 1 Consider the harmonic oscillator system

$$X' = \begin{pmatrix} 0 & 1 \\ -k & -b \end{pmatrix} X,$$

where  $b \geq 0$ ,  $k > 0$  and  $m = 1$ .

- (a) For which values of  $b, k$  does this system have complex eigenvalues?
- (b) Describe the motion of the mass when the mass is released from initial position  $x = 1$  with zero velocity in the case of part (a).

Problem 2

Consider the system

$$X' = \begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix} X.$$

Find the general solution of this system.

Problem 3

Consider the one-parameter family of linear systems given by

$$X' = \begin{pmatrix} 2a & 1+a \\ 1-a & 0 \end{pmatrix} X,$$

where  $a$  is a real parameter. Sketch the different phase portraits that arise as  $a$  varies.

Problem 4

Identify regions in the  $ab$ -plane where the system

$$X' = \begin{pmatrix} a & b \\ b & a \end{pmatrix} X$$

has similar phase portraits.