

Problem 1

Consider the systems $X' = AX$ and $Y' = BY$, where

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

- (a) Find the flow for each system.
- (b) Prove that $H(x, y) = (x, -y)$ provides a conjugacy between the flows you found in the previous part.

For the following three problems:

- (a) put the matrices A in canonical form,
- (b) sketch the phase portrait (or cross sections of it) for the system corresponding to the canonical form, and
- (c) find the general solution of the original system $X' = AX$

Problem 2

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Problem 3

$$A = \begin{pmatrix} 1 & 8 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

Problem 4

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$