

Problem 1

Consider the systems $X' = AX$ and $Y' = BY$, where

$$A = \begin{pmatrix} 0 & 1 \\ -4 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}.$$

- (a) Find the flow for each system.
- (b) Find an explicit conjugacy between the flows you found in the previous part.

Problem 2

Prove that any two linear systems with the same eigenvalues $\pm i\beta$, $\beta \neq 0$ are conjugate. What happens if the systems have eigenvalues $\pm i\beta$ and $\pm i\gamma$, with $\gamma = -\beta$?

Problem 3

Consider the system

$$X' = \begin{pmatrix} a & 0 & b \\ 0 & b & 0 \\ -b & 0 & a \end{pmatrix} X,$$

depending on the two real parameters a, b .

- (a) Sketch the regions in the ab -plane where this system has different types of phase portraits.
- (b) Draw examples of phase portraits for the canonical systems that correspond to the systems of the previous part.