MATH 489, Section C14, HW 5. Due date: 03/18/09.

Problem 1

Consider $X' = AX$ where A is an 4×4 matrix in canonical form with eigenvalues $\pm i$ and $-1 \pm i$. Describe this flow.

Problem 2

Consider the setting where we have a spring with constant k_1, coupled to a mass m_1, coupled to a spring with constant k_2, coupled to a mass m_2, which is finally coupled to another spring with constant k_1. Let x_j denote the displacement of each mass from its rest position, and assume that both masses are equal to 1. The differential equations for these coupled oscillators are then given by

$$\begin{aligned} x_1'' &= -(k_1 + k_2)x_1 + k_2 x_2 \\ x_2'' &= k_2 x_1 - (k_1 + k_2)x_2. \end{aligned}$$

(a) Write these equations as a first order system and find its eigenvalues.

(b) Let $\omega_1 = \sqrt{k_1}$ and $\omega_2 = \sqrt{k_1 + 2k_2}$. What can be said about the periodicity of the solutions relative to ω_j?

Problem 3

Let A be an $n \times n$ matrix. Show that the Picard method for solving $X' = AX$, $X(0) = X_0$ gives the solution $\exp(tA)X_0$.