

MATH 489, Section C13, HW 6. Due date: 04/10/09.

For each of the following two nonlinear systems,

(a) Find all of the equilibrium points and describe the behavior of the associated linearized system.

(b) Describe the phase portrait of the nonlinear system.

(c) Does the linearized system accurately describe the local behavior near the equilibrium points?

Problem 1

$$\begin{aligned}x' &= x(x^2 + y^2) \\y' &= y(x^2 + y^2)\end{aligned}$$

Problem 2

$$\begin{aligned}x' &= x + x^2 \\y' &= y\end{aligned}$$

Problem 3

Find a global change of coordinates that linearizes the system

$$\begin{aligned}x' &= x - y^2 \\y' &= y \\z' &= -2z - y^2.\end{aligned}$$

Problem 4

Consider the system

$$\begin{aligned}x' &= x^2 + y \\y' &= x - y + a\end{aligned}$$

where  $a$  is a parameter.

(a) Find all equilibrium points and compute the linearized equation at each.

(b) Describe the behavior of the linearized system at each equilibrium point.

(c) Describe any bifurcations that occur.