

MATH 489, Section C14, HW 6. Due date: 04/10/09.

For each of the following two nonlinear systems,

(a) Find all of the equilibrium points and describe the behavior of the associated linearized system.

(b) Describe the phase portrait of the nonlinear system.

(c) Does the linearized system accurately describe the local behavior near the equilibrium points?

Problem 1

$$\begin{aligned}x' &= \sin x \\y' &= \cos y\end{aligned}$$

Problem 2

$$\begin{aligned}x' &= y^2 \\y' &= y\end{aligned}$$

Problem 3

Consider the first-order differential equation  $x' = f_a(x)$  for which  $f_a(x_0) = 0$  and  $f'_a(x_0) \neq 0$ . Prove that the differential equation  $x' = f_{a+\epsilon}(x)$  has an equilibrium point  $x_0(\epsilon)$  where  $\epsilon \rightarrow x_0(\epsilon)$  is a smooth function satisfying  $x_0(0) = x_0$ .