MATH 441 SECTION C13 Review Problems for the Final Exam

Coverage: Sections 1.1-1.3, 2.1-2.6, 2.8, 3.1-3.9, 4.1-4.4, 5.1-5.8.

Problem 1

Consider the IVP $y' = 3y^{2/3}$ and $y(0) = 1$. Suppose that $a > 0$ and y is a solution to the IVP on $(-a, a)$ such that $y(t) \neq 0$ for all $t \in (-a, a)$. Solve the problem explicitly for y and use the expression of y to find a bound for a. If $a > 1$, show that there is not a unique solution of the IVP on $(-a, a)$.

Problem 2

State the existence and uniqueness theorem for $y' = f(t, y)$ and $y(0) = 0$, where $f(t, y)$ and $\frac{\partial f}{\partial y}(t, y)$ are defined and continuous for all $t, y \in R$. What are the main steps of the prrof?

Problem 3

Consider the DE $y'' + p(t)y' + q(t)y = 0$. Assume that $p(t)$ and $q(t)$ are continuous on R. Suppose y_1 and y_2 are linearly independent solutions of the DE on R and let $W(t)$ be their Wronskian. Use the existence and uniqueness theorem for second order linear differential equations to explain why $W(1) \neq 0$. Use Abel's theorem to give a second explanation of the fact that $W(1) \neq 0$.

Problem 4

Find the general solution of the DE $t^2y'' - 2y = 3t^2 - 1$ for $t > 0$. Hint: the DE is an Euler equation. Use the method of variation of parameters to find a particular solution.

Problem 5

Find the general solution of the DE $y''' - 3y'' + 4y' - 2y = e^x$.

Problem 6

Consider the DE $y'' - 2xy' + 10y = 0$. Solve it by means of a power series. Determine the radius of convergence and the reccurence formula, if possible. Can you find a polynomial solution?

Problem 7

Consider the DE $x^2y'' + (3x - 1)y' + y = 0$ on $x > 0$. Does this DE have a solution of the form $y = x^r \sum_{n=0}^{\infty}$ in which $a_0 \neq 0$ and the power series $\sum_{n=0}^{\infty}$ has a positive radius of convergence? If yes, find it. If not, does this violate the existence and uniqueness theorem for second order linear equations?

Problem 8

Consider the Bessel Equation of order ν, $x^2y'' + xy' + (x^2 - \nu^2)y = 0$, $x > 0$ where ν is real and positive. Find two linearly independent solutions, in the case where 2ν is not an integer.

Problem 9

Study the problems on the review sheets for exams 1, 2, and 3.