

QUIZ 10

Name:

UIN:

1. Sketch the parabolas with equation

(a) $x^2 = 2x - 8y + 1$, and

(b) $y^2 = 2y - 8x + 1$.

Show and label its vertex, focus, and directrix.

Solution: (a) first rewrite the equation in a more convenient form

$$x^2 = 2x - 8y + 1 \quad \Leftrightarrow \quad x^2 - 2x + 1 = -8y + 2 \quad \Leftrightarrow \quad (x - 1)^2 = -8\left(y - \frac{1}{4}\right).$$

From this, one can see that its vertex has coordinates $(1, \frac{1}{4})$, its focus is the point $(1, -\frac{7}{4})$, and its directrix has equation $y = \frac{9}{4}$. The parabola opens downwards.

(b) This part follows in exactly the same way after one has interchanged the roles of x and y .

2. Find the equation of the ellipses with foci

(a) $(1, 1)$, $(1, 5)$, and minor axis of length 6, and

(b) $(1, 1)$, $(5, 1)$, and minor axis of length 6.

Solution: (a) since the foci lie on the line $x = 1$, it follows that the center also lies on this line, and it holds that $2c = 5 - 1 = 4 \Rightarrow c = 2$. Hence, the center is the point $(1, 3)$. The minor axis has length 6, so one obtains that $2b = 6 \Rightarrow b = 3$. Using the values of b and c , one can calculate $a^2 = b^2 + c^2 = 13$. Therefore, the equation of the ellipse is

$$\frac{(y - 3)^2}{13} + \frac{(x - 1)^2}{9} = 1.$$

(b) Again, this part follows in the same way after the roles of x and y have been interchanged.