QUIZ 7

Name: UIN:

Find the Taylor polynomial of the functions $f(x) = \frac{1}{x+2}$ and $f(x) = \frac{1}{x+3}$ using $a = 0$ and $n = 3$.

Solution:

$$\begin{aligned} f(x) &= (x+2)^{-1} &\Rightarrow\quad f(0) &= \frac{1}{2} \\ f'(x) &= (-1)(x+2)^{-2} &\Rightarrow\quad f'(0) &= -\frac{1}{4} \\ f''(x) &= (-1)(-2)(x+2)^{-3} &\Rightarrow\quad f''(0) &= \frac{1}{4} \\ f'''(x) &= (-1)(-2)(-3)(x+2)^{-4} &\Rightarrow\quad f'''(0) &= -\frac{3}{8}. \end{aligned}$$

Hence, the Taylor polynamial is

$$P_3(x) = \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3.$$

Similarly, for $f(x) = \frac{1}{x+3}$ we obtain

$$P_3(x) = \frac{1}{3} - \frac{1}{9}x + \frac{1}{27}x^2 - \frac{1}{81}x^3.$$

Find the Maclaurin series of the functions $f(x) = \sin(3\,x)$ and $f(x) = \sin(x^3)$ by substituting in the known series for $\sin x$.

Solution: The Maclaurin series for $\sin x$ is

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2\,n+1}}{(2\,n+1)!}.$$

Therefore,

$$\sin(3\,x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2\,n+1} x^{2\,n+1}}{(2\,n+1)!}$$

and

$$\sin x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6\,n+3}}{(2\,n+1)!}$$