QUIZ 9

Name: UIN:

1. Use POWER SERIES rather than L'Hopital's rule to evaluate the limits

$$\lim_{x\to 0} \frac{e^x - 1 - \sin x}{x^2},$$

and

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x}.$$

Solution: we will use the power series

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + ...$$

and

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{5!} - ....$$

Then,

$$\begin{aligned}
\lim_{x\to 0} \frac{e^x - 1 - \sin x}{x^2} &= \lim_{x\to 0} \frac{1 + x + \frac{x^2}{2} + \frac{x^3}{6} + ... - 1 - (x - \frac{x^3}{6} + ...)}{x^2} \\
&= \lim_{x\to 0} \frac{\frac{x^2}{2} + \frac{x^3}{3} + ...}{x^2} \\
&= \lim_{x\to 0} \left(\frac{1}{2} + \frac{x}{3} + ...\right) \\
&= \frac{1}{2}
\end{aligned}$$

For the second limit we have,

$$\begin{aligned}
\lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x} &= \lim_{x\to 0} \frac{1 + x + \frac{x^2}{2} + \frac{x^3}{6} + ... - (1 - x + \frac{x^2}{2} - \frac{x^3}{6} + ...)}{x - \frac{x^3}{6} + \frac{x^5}{5!} - ...} \\
&= \lim_{x\to 0} \frac{2\,x + \frac{2\,x^3}{3} + ...}{x - \frac{x^3}{6} + \frac{x^5}{5!} - ...} \\
&= \lim_{x\to 0} \frac{2 + \frac{2\,x^2}{3} + ...}{1 - \frac{x^2}{6} + \frac{x^4}{5!} - ...} \\
&= 2
\end{aligned}$$

2. Write the equation of the circles with center $(2, -2)$ that is tangent to the line $y = 2$ and with center $(1, -1)$ that is tangent to the line $x = 2$.

Solution: it is easy to see that the radius of the circle equals the distance of its center from the tangent line $y = 2$. This distance is $r = 4$, so the equation of the circle is

$$(x - 2)^2 + (y + 2)^2 = 16.$$

Similarly for the second circle, its radius equals the distance of the center from the tangent line $x = 2$, which is $r = 1$. Therefore, the equation of the circle is

$$(x - 1)^2 + (y + 1)^2 = 1.$$