REVIEW QUESIONS FOR THE FINAL EXAM

MATH 230, Section BL1, Fall 2005

1. Find the arc length of the polar curve $r = 1 - \cos\theta$, $0 \le \theta \le \frac{\pi}{2}$.

2. Sketch the graph of the equation $\frac{(x-1)^2}{9} + \frac{(y-3)^2}{16} = 1$. Find its foci, center, and vertices.

3. Find the arc length of the parametric curve $x = 3\cos^3 t$, $y = 3\sin^3 t$, $0 \le t \le \frac{\pi}{2}$.

4. Solve the initial value problem $\frac{dy}{dx} = \frac{3}{y^2}$, $y(0) = 1$.

5. Determine whether or not the improper integrals below converge or diverge. Justify your answer. Evaluate those that converge.

$$\int_{-\infty}^{\infty} \frac{x}{(x^2+4)^{\frac{3}{2}}}\,dx \qquad \int_0^5 \frac{1}{x-5}\,dx$$

6. Evaluate the integrals

$$\int \frac{x^2}{\sqrt{1-x}}\,dx \qquad \int \frac{x+1}{x^3-x^2}\,dx \qquad \int \frac{1}{(9-x^2)^{\frac{3}{2}}}\,dx$$

7. Estimate within 0.1, $\quad e^{0.8}$.

8. Find the interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{x^n}{n} \qquad \sum_{n=1}^{\infty} n\frac{(x-1)^n}{6^n}$$

9. Determine whether the series below converge or diverge:

$$\sum_{n=1}^{\infty} \frac{2+\cos n}{\sqrt{n+1}} \qquad \sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n$$

10. Determine whether the series below converge absolutely, conditionally or diverge. Justify your answer.

$$\sum_{n=1}^{\infty} \frac{n!}{(-2)^n} \qquad \sum_{n=1}^{\infty} (-1)^n\left(\sqrt{n+1} - \sqrt{n}\right)$$

Answers:

1. $4 - 2\sqrt{2}$
2. center $(1, 3)$, vertices $(1, 7)$, $(1, -1)$, $(4, 3)$, and $(-2, 3)$, foci $(1, 3 + \sqrt{7})$ and $(1, 3 - \sqrt{7})$
3. $\frac{9}{2}$
4. $y = (9\,x + 1)^{\frac{1}{3}}$
5. 0, diverges
6. $-\frac{2}{15}\sqrt{1 - x}\,(3\,x^2 + 4\,x + 8) + C$, $-2\ln|x| + \frac{1}{x} + 2\ln|x + 1| + C$, $\frac{x}{9\sqrt{9 - x^2}} + C$
7. 2.2
8. $[-1, 1)$, $(-5, 7)$
9. diverges, converges
10. diverges, converges conditionally